Received: *April 26, 2000*, Accepted: *Juin 4, 2000*

Quantization of four-form fluxes and dynamical neutralization of the cosmological constant

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Abstract: A four-form gauge flux makes a variable contribution to the cosmological constant. This has often been assumed to take continuous values, but we argue that it has a generalized Dirac quantization condition. For a single flux the steps are much larger than the observational limit, but we show that with multiple fluxes the allowed values can form a sufficiently dense 'discretuum'. Multiple fluxes generally arise in M-theory compactifications on manifolds with non-trivial three-cycles. In theories with large extra dimensions a few four-forms suffice; otherwise of order 100 are needed. Starting from generic initial conditions, the repeated nucleation of membranes dynamically generates regions with λ in the observational range. Entropy and density perturbations can be produced.

Keywords: M-Theory, p-branes, Superstring Vacua, Cosmology of Theories beyond the SM.

JHEP06(2000)006

Contents

1. Introduction

The cosmological constant problem is one of the central challenges in quantum gravity, and it continues to provoke novel and interesting ideas.[1] In some approaches, the cosmological 'constant' becomes a dynamical variable: it is locally constant but has a continuous range of allowed values, with the effective value in our universe determined by some dynamical principle.

There are several mechanisms by which the cosmological constant can become a dynamical variable. One is through the existence of a four-form field strength [3, 4, 5]. The equation of motion requires that such a field strength be constant, so it has no local dynamics but contributes a positive energy density, which can cancel a cosmological constant coming from other sourcs if the latter is negative. A second

[1] For a classic survey of various approaches see Weinberg's review [1]. A brief survey of recent ideas is given in ref. [2].

mechanism is fluctuations of the topology of spacetime (wormholes). Under a plausible interpretation of the path integral for quantum gravity these convert all constants of Nature into dynamical variables [6, 7], though there is serious doubt as to whether this effect exists in a real theory of quantum gravity. A third mechanism is the existence of naked singularities in compactified dimensions [8, 9], where the undetermined boundary conditions at the singularity become a variation of the effective four dimensional lagrangian.

In this paper we will discuss certain aspects of the four-form idea, though at the end we will note that very similar considerations may apply to the naked singularity. Our first point is that the four-form field strength, although usually assumed to take continuous values, is in fact quantized. This quantization might be evaded in a purely four-dimensional theory, but is certainly necessary when gravity is embedded in a higher-dimensional theory such as M-theory. The size of the quantum is fixed by microscopic physics, and so the spacing of energy densities is enormous compared to the actual value, or bound, on the cosmological constant. Therefore the four-form cannot play the assumed role of producing a small cosmological constant.

This is discouraging, but there is a variant of the four-form idea which has some interesting features. Typical M-theory compactifications have extra four-form field strengths, arising from nontrivial three-cycles in the compactification. If there are several four-form field strengths, with incommensurate charges, then the allowed cosmological constants may form a closely spaced 'discretuum', with one or more values in the experimentally allowed range. The universe can reach such a value, starting from a larger density, through a series of domain wall nucleations. This resembles an idea of Brown and Teitelboim [10, 11], but has some unique and attractive features. In particular, there is a plausible mechanism for heating the universe after nucleation produces a small cosmological constant.

A second complication is that in higher dimensional theories there are in general moduli, and the four-form does not produce a constant energy density but rather a potential for the moduli. The analysis of the four-form flux therefore cannot be separated from the consideration of the stability of the compactification. This is a difficult issue for a number of reasons, and aside from a brief discussion will sidestep it by working in an artificial, model where the charges are frozen in incommensurate ratios. Although this is rather optimistic, it may be that some features of the cosmology that we find will survive in more realistic circumstances.

In section 2 we review the physics of four-form fluxes and explain how a generalized Dirac quantization condition constrains the value of the four-form flux. We then investigate the level spacing in a theory of many four-forms, as generally arise in M-theory compactifications. We find that the discretuum is sufficiently dense if there is a membrane charge of order 10^{-1} and a large number of fluxes, say 100. The large dimension scenario [12] produces much smaller charges, and can lead to a sufficiently dense discretuum for as few as four fluxes.

In section 3 we discuss the resulting cosmology. We review the Brown-Teitelboim scenario, in which a cosmological constant is neutralized by nucleation of membranes. We then extend this to multiple four-forms. If the flux density is initially large, so that the cosmological constant is positive, then one obtains a picture much like eternal inflation, where the cosmological constant takes different values in different expanding bubbles. De Sitter thermal effects provide a natural solution to one of the serious problems of the Brown-Teitelboim idea. The inflaton can be stabilized in the inflationary part of its potential until the nucleation reduces the cosmological constant to near zero, at which point it begins to roll. This is possible because with multiple four-forms the individual jumps in the cosmological constant can be quite large. In the end the observed cosmological constant is small for anthropic reasons, but in the weakest sense: we have a universe with different cosmological constants in different regions, and with galaxies only in regions of small cosmological constant. In many respects our picture resembles an idea of Banks [13]. Another example of a discretuum is the irrational axion [14, 15]. While this work was being completed we learned that Feng, March-Russell, Sethi, and Wilczek are also considering extensions of the mechanism of Brown and Teitelboim.

2. Four-form quantization

2.1 Four-form energetics

We first review the basic physics of four-form field strengths. For antisymmetric tensor fields, the language of forms is used when convenient; this is indicated by bold face ($\mathbf{F_4}$). Normal fonts are used for index notation, or when index notation is implied, e.g. $F_4^2 = F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}$.

The action for gravity with a bare vacuum energy λ_{bare} plus four-form kinetic term is

$$S = \int d^4x\, \sqrt{-g}\Big(\frac{1}{2\kappa_4^2}R - \lambda_{\text{bare}} - \frac{Z}{2\cdot 4!}F_4^2\Big) + S_{\text{branes}}\,, \tag{2.1}$$

where $\mathbf{F_4} = \mathbf{dA_3}$. We include a general normalization constant Z in the kinetic term for later convenience. Certain boundary terms must be added to this action. They do not affect the equations of motion and will not be prominent in the remainder of this paper. However, they are crucial for the correct evaluation of the on-shell action when physical quantities are measured on an equal time hypersurface Σ. The usual Gibbons-Hawking term [16] is given by

$$S_{\text{GH}} = \frac{1}{\kappa_4^2}\int_\Sigma d^3x\, \sqrt{h}K\,. \tag{2.2}$$

For the four-form field the following boundary term must be included to obtain stationary action under variations that leave F fixed on the boundary [17]:

$$S_{\text{DJ}} = \frac{Z}{3!}\int d^4x\, \partial_\mu\left(\sqrt{-g}F^{\mu\nu\rho\lambda}A_{\nu\rho\lambda}\right). \tag{2.3}$$

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On shell its value is negative twice the F^2 contribution in the volume term of the action. This removes the apparent discrepancy [18] between the cosmological constant in the on-shell action and in the equations of motion.

Ignoring the brane sources (we will consider them shortly), the four-form equation of motion is $\partial_\mu \left(\sqrt{-g}\, F^{\mu\nu\rho\sigma}\right) = 0$, with solution

$$F^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}\,, \tag{2.4}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor and c is any constant. Thus there is no local dynamics. One has $F_4^2 = -24c^2$, and so the on-shell effect of the four-form is indistinguishable from a cosmological constant term. The hamiltonian density is given by

$$\lambda = \lambda_{\rm bare} - \frac{Z}{48}F_4^2 = \lambda_{\rm bare} + \frac{Zc^2}{2}\,. \tag{2.5}$$

Only λ is observable: $\lambda_{\rm bare}$ and the four-form cannot be observed separately in the four-dimensional theory. Therefore, the bare cosmological constant can be quite large. For example, it might be on the Planck scale or on the supersymmetry breaking scale. In order to explain the observed value of the cosmological constant, $\lambda_{\rm bare}$ must be very nearly cancelled by the four-form contribution.

2.2 Four-form quantization

In the original work [5], and in many recent applications, it as assumed that the constant c can take any real value, thus cancelling the bare cosmological constant to arbitrary accuracy. However, we are asserting that the value of c is quantized. Since this is somewhat counterintuitive, let us first discuss two things that the reader might think we are saying, but are not.

First, if there is a gravitational instanton, a euclidean four-manifold X, then it is natural to expect that the integral of the euclidean four-form over X is quantized,

$$\int_X \mathbf{F_4} = \frac{2\pi n}{e}\,, \qquad n \in \mathbb{Z}\,. \tag{2.6}$$

This is the generalized Dirac quantization condition [19, 20, 21, 22]. It arises from considering the quantum mechanics of membranes, which are the natural objects to couple to the potential $\mathbf{A_3}$,

$$S = e\int_W \mathbf{A_3}\,, \tag{2.7}$$

with e the membrane's charge and W its world-volume. The condition that membrane amplitudes be single-valued then implies the quantization (2.6). This is true, but we are asserting something in addition: that the actual *value* of $\mathbf{F_4}$ (or, more precisely, c) is quantized, in addition to the integral.

Of course, the inclusion of membranes means that c is no longer globally constant, as the membranes are sources for $\mathbf{F_4}$. The value of c jumps across a membrane,

$$\Delta c = \frac{e}{Z} \,. \tag{2.8}$$

The total change in c due to nucleation of any number of membranes is then a multiple of e/Z.

However, it is not this change that we are asserting is quantized, but the actual value:

$$c = \frac{en}{Z} \,, \qquad n \in \mathbb{Z} \,. \tag{2.9}$$

This may seem surprising, but in fact is quite natural. String theory has the satisfying property that for every gauge field there exist both electric and magnetic sources. This implies a quantization condition both for the field strength and its dual. The dual of a four-form is a zero-form,

$$*\mathbf{F_4} = \mathbf{F_0} \,. \tag{2.10}$$

A zero-form is naturally integrated over a zero-dimensional manifold, which is to say that it is evaluated at a point. The generalized Dirac condition is that this be quantized, which is precisely eq. (2.9):

$$\mathbf{F_0} = \frac{en}{Z} \,, \qquad n \in \mathbb{Z} \,. \tag{2.11}$$

The quantizations (2.6) and (2.11) are in just the usual relation [22] for n-form and $(d-n)$-form field strengths in d spacetime dimensions.

Although natural, it is not clear that the quantization of $\mathbf{F_0}$ is necessary. The quantization of $\mathbf{F_4}$ arises from the consistency of the quantum mechanics of 2-branes, but that of $\mathbf{F_0}$ would come from the quantum mechanics of (-2)-branes, and it is not clear what this should be. Further, there is the example of the Schwinger model, where the non-integer part of $\mathbf{F_0}$ is just the θ-parameter, which can take any real value.

Nevertheless, the quantization condition (2.11) is necessary when the four-dimensional theory is embedded in string theory.[2] Consider for example the compactification of M-theory on a seven-manifold K. We begin with the eleven-dimensional action

$$S = 2\pi M_{11}^9 \int d^{11}X \sqrt{-g_{11}} \left(R - \frac{1}{2 \cdot 4!} F_4^2 \right) + S_{\text{branes}} \,, \tag{2.12}$$

where we omit the Chern-Simons and fermion terms, which will play no role. With this normalization the M2-brane tension and charge are $2\pi M_{11}^3$, and the M5-brane

[2]This observation grew out of ref. [23], where quantization of a top-form (or zero-form) field strength first appeared.

charge and tension are $2\pi M_{11}^6$.[3] The M5-brane couples to $\mathbf{A_6}$,

$$2\pi M_{11}^6 \int_W \mathbf{A_6}\,, \tag{2.13}$$

where W is the M5 world-volume, and

$$\mathbf{dA_6} = \mathbf{F_7} = *_{11}\mathbf{F_4}\,, \tag{2.14}$$

where a subscript is used to distinguish the dual in eleven-dimensions from that in four dimensions. By the generalized Dirac quantization it follows that

$$2\pi M_{11}^6 \int_K \mathbf{F_7} = 2\pi n\,, \qquad n \in \mathbb{Z}\,. \tag{2.15}$$

Now reduce to four dimensions. The eleven dimensional $\mathbf{F_4}$ reduces directly to a four dimensional $\mathbf{F_4}$, with action

$$S = V_7 2\pi M_{11}^9 \int d^4x\,\sqrt{-g}\left(R - \frac{1}{2\cdot 4!}F_4^2\right) + S_{\text{branes}}\,, \tag{2.16}$$

where V_7 is the volume of K. Further, the condition (2.15) becomes

$$\mathbf{F_0} = \frac{n}{M_{11}^6 V_7}\,, \qquad n \in \mathbb{Z}\,. \tag{2.17}$$

That is,

$$(2\kappa_4^2)^{-1} = Z = 2\pi M_{11}^9 V_7\,. \tag{2.18}$$

The quantization (2.17) matches that found in eq. (2.11) with $e = 2\pi M_{11}^3$, which is just the M2-brane tension.

2.3 Discussion

The quantization that we have found rules out the precise cancellation of the cosmological constant that has been assumed in many discussions. Brown and Teitelboim [10, 11] considered the approximate neutralization of the cosmological constant by a field strength taking discrete values (see also Abbott [25] for a closely related idea). In order that this be natural, the spacing between allowed values of λ must be of order the observational bound. Since $d\lambda/dn = 2ne^2/Z$ and $n_{\text{final}} \approx \sqrt{|\lambda_{\text{bare}}|Z}/e$, the final value of λ will lie within observational bounds only if

$$e|\lambda_{\text{bare}}|^{1/2} Z^{-1/2} < 10^{-120}\kappa_4^{-4}\,. \tag{2.19}$$

Using the results above for e and Z, the left-hand side (dropping 2π's) is

$$|\lambda_{\text{bare}}|^{1/2}\kappa_4^{1/3} V_7^{-1/3} \sim |\lambda_{\text{bare}}|^{1/2}\kappa_4 M_{11}^3\,. \tag{2.20}$$

[3] For a review see ref. [24].

The step size is minimized in the low-energy string scenario [12], where $\lambda_{\rm bare}$ and M_{11} are both TeV-scale, but even in this case it is far too large, $10^{-75}\kappa_4^{-4}$.

This is the 'gap problem': the Brown-Teitelboim mechanism requires an energy spacing which is infinitesimal compared to the scales of microphysics. In the next subsection we will consider compactification with multiple four-forms, which can reduce the step size to an acceptable value.

Because the compactification volume V_7 is a dynamical quantity and not a fixed parameter, the four-form energy density is not a constant but a potential for V_7. In a realistic compactification this must be stabilized, and the energetics of the four-form fluxes will enter into the stabilization.[4] Thus the volume V_7 itself depends on n, and so the effective cosmological constant has additional n-dependence beyond that included above. For convenience we will in the rest of this paper ignore this effect, treating the geometry as fixed.

It should be noted that the allowed flux actually depends additively on the values of flat background gauge potentials[5] — these are just stringy generalizations of the Schwinger model θ-parameter. As these backgrounds vary the flux can take arbitrary real values. This does not, however, restore the original continuously variable cosmological constant, because these background potentials are moduli and not parameters. As with the compactification geometry, these background moduli must eventually be stabilized and so the fluxes will in fact take discrete values.

2.4 Multiple four-forms

General compactifications actually give rise to several four-form fluxes, and this can solve the gap problem. Let there be J such fluxes, with

$$\lambda = \lambda_{\rm bare} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2 \,. \tag{2.21}$$

The question is whether there exists a set of n_i such that

$$2|\lambda_{\rm bare}| < \sum_{i=1}^{J} n_i^2 q_i^2 < 2(|\lambda_{\rm bare}| + \Delta\lambda)\,, \tag{2.22}$$

where $\Delta\lambda$ corresponds to the observational bound, roughly 10^{-120} in Planck units. This can be visualized in terms of a J-dimensional grid of points, spaced by q_i and labeled by n_i (see figure 1).

Consider a sphere of radius $r = |2\lambda_{\rm bare}|^{1/2}$ centered at $n_i = 0$. If one of the points $(n_1, n_2, \ldots, n_J)$ is sufficiently close to the sphere, the field configuration corresponding to this point will lead to an acceptable value of the cosmological constant.

[4] See for example the discussions [26, 27].

[5] We thank E. Witten for pointing this out.

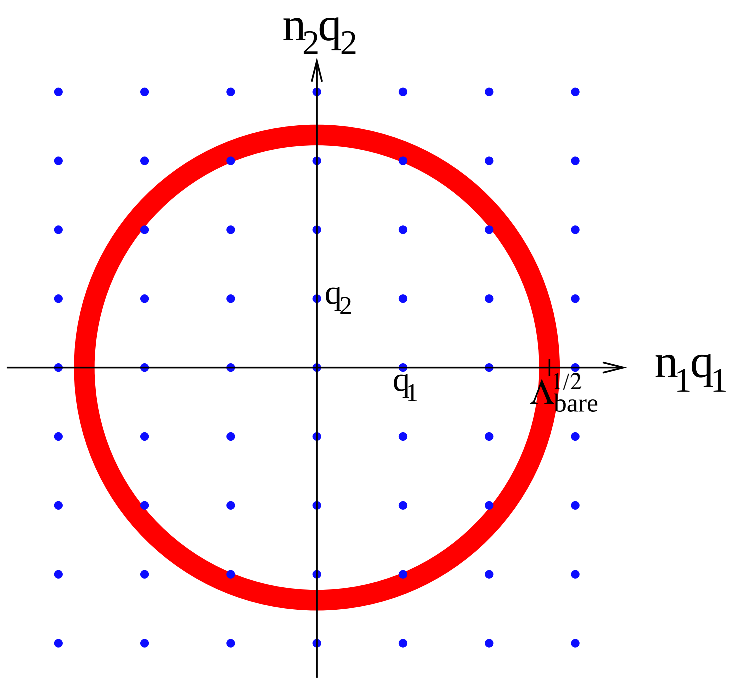

Figure 1: The allowed values of the four-form energy density are given by the radius-squared of points in the grid, whose dimension is the number of four-forms J. The spacing in direction i is q_i. The negative of the bare cosmological constant corresponds to a $(J-1)$-dimensional sphere, and cancellation is possible if there is at least one grid point sufficiently close to the sphere.

More precisely, one should think of a thin shell, whose width encodes the width of the observational range,

$$\Delta r = |2\lambda_{\text{bare}}|^{-1/2}\Delta\lambda\,. \tag{2.23}$$

We need at least one point to lie within the shell. As we will discuss, there may be large degeneracies — let the typical degeneracy be D. The volume per D grid points must then be less than the volume of the shell, $\omega_{J-1}r^{J-1}\Delta r$, where the area of a unit sphere is $\omega_{J-1} = 2\pi^{J/2}/\Gamma(J/2)$. Thus

$$\prod_{i=1}^{J} q_i \lesssim \frac{\omega_{J-1}}{D}|2\lambda_{\text{bare}}|^{J/2-1}\Delta\lambda\,, \tag{2.24}$$

or

$$\frac{D}{\omega_{J-1}}\prod_{i=1}^{J}\frac{q_i}{|2\lambda_{\text{bare}}|^{1/2}} \lesssim \frac{\Delta\lambda}{|2\lambda_{\text{bare}}|}\,. \tag{2.25}$$

In other words, the typical spacing of the spectrum of the cosmological constant in a model with given J, e_i, and λ_{bare} will be given by

$$\Delta\lambda_{\text{min}} = \frac{D\prod_{i=1}^{J} q_i}{\omega_{J-1}|2\lambda_{\text{bare}}|^{J/2-1}}\,. \tag{2.26}$$

An important feature of this result is that that the q_i need not be exceedingly small if there are more than two four-form fields. In order to achieve a small λ, it is sufficient that there be a discrepancy between the magnitude of $\lambda_{\rm bare}$ and that of the charges. For fixed charges, the task of cancellation actually becomes easier, the larger the bare cosmological constant. This can be understood from figure 1. The larger the shell, the more points it will contain.[6] The results (2.24) to (2.26) treat the n_i as essentially continuous, and break down if any of the q_i exceed $J^{-1/2}|2\lambda_{\rm bare}|^{1/2}$. In this case the flux associated with q_i should simply be ignored.

For illustration suppose that $\lambda_{\rm bare}$ is at the Planck scale, $(\sqrt{2}\kappa_4)^4\lambda_{\rm bare}\sim 1$, that the number of four-forms is 100, a number which is large but not unrealistic, and that D is small. Then the inequality (2.25) implies that the typical charge must be of order $10^{-1.6}$ in Planck units; note that q_i is a mass-squared, so we should perhaps measure the smallness by the square root, $10^{-0.8}\sim 1/6$. However, the assumption of no degeneracy is rather optimistic, as we will discuss in the next subsection.

2.5 M-theory compactification

Consider the compactification of M-theory on a general manifold K. The total number of fluxes is $J = N_3+1$, where N_3 is the number of nontrivial three-cycles of K. For each nontrivial three-cycle C_i there is a harmonic three-form $\omega_{\mathbf{3},i}$, and the seven-form field strength can be expanded

$$\mathbf{F_7} = \frac{1}{M_{11}^3}\sum_{i=1}^{N_3}\mathbf{F}_{\mathbf{4},i}(x)\wedge\omega_{\mathbf{3},i}(y) + *\mathbf{F}_{\mathbf{4},N_3+1}(x)\wedge\epsilon_{\mathbf{7}}(y)\,. \tag{2.27}$$

Here $\epsilon_{\mathbf{7}}$ is the volume form on K, so that $\mathbf{F}_{\mathbf{4},N_3+1}$ is the flux discussed previously, obtained simply by reduction of the eleven-dimensional flux. Coordinates have been labeled as follows:

$$(X^0,\ldots,X^{11}) = (x^0,\ldots,x^3,y^1,\ldots,y^7)\equiv(x^\mu,y^m)\,. \tag{2.28}$$

Associated to each flux $\mathbf{F}_{\mathbf{4},i}$ is a four-dimensional domain wall (membrane), obtained by wrapping three legs of the M5-brane on C_i.

Let us illustrate this by a simple model, in which K is simply a seven-torus with flat internal metric δ_{mn}, and with y^m identified with period $2\pi r_m$; then $V_7 = \prod_{m=1}^7(2\pi r_m)$. There is one three-cycle C_i for each unordered triplet (m_i,m_i',m_i''), or $\binom{7}{3}=35$ in all. The volume and three-form associated with C_i are

$$V_{3,i} = (2\pi)^3 r_{m_i}r_{m_i'}r_{m_i''}\,,\qquad \omega_{\mathbf{3},i} = \frac{1}{V_{3,i}}\mathbf{dy}^{m_i}\wedge\mathbf{dy}^{m_i'}\wedge\mathbf{dy}^{m_i''}\,. \tag{2.29}$$

[6] Note, however, that the radius of the shell in figure 1 represents not $|2\lambda_{\rm bare}|$, but the square root of $|2\lambda_{\rm bare}|$. This is why one cannot recognize in figure 1 the need for the charges q_i to be incommensurate, a fact that is immediately clear from eq. (2.21). It is also the reason why increasing $|\lambda_{\rm bare}|$ has no beneficial effect in the case of $J=2$. For fixed $\Delta\lambda$, the shell gets thinner as one increases its radius. If $J=2$, this precisely compensates for the increase of the shell radius, and the volume remains constant.

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The four-dimensional action is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa_4^2} R - \lambda_{\rm bare} - \frac{1}{2 \cdot 4!} \sum_{i=1}^{N_3+1} Z_i F_{4,i}^2 \right) + S_{\rm branes} \,. \tag{2.30}$$

Here

$$Z_i = \frac{2\pi M_{11}^3 V_7}{V_{3,i}^2} \quad (i \leq N_3) \,, \qquad \frac{1}{2\kappa_4^2} = Z_{N_3+1} = 2\pi M_{11}^9 V_7 \,. \tag{2.31}$$

The bare cosmological constant has been added by hand in this model. In a real compactification, negative energy density can arise from positive scalar curvature or an orientifold plane, for example. The tension of a membrane wrapped on C_i is

$$\tau_i = 2\pi M_{11}^6 V_{3,i} \quad (i \leq N_3) \,, \qquad \tau_{N_3+1} = 2\pi M_{11}^3 \,. \tag{2.32}$$

Its coupling to the j'th three-form potential is

$$e_{i,j} = e\delta_{ij} \,, \qquad e = 2\pi M_{11}^3 \,. \tag{2.33}$$

The quantization condition is

$$\mathbf{F}_{\mathbf{0},i} = \frac{en_i}{Z_i} \,, \tag{2.34}$$

and the effective cosmological constant is

$$\lambda = \lambda_{\rm bare} + \sum_{i=1}^{N_3+1} \frac{e^2 n_i^2}{2Z_i} \,, \tag{2.35}$$

so that

$$q_i = eZ_i^{-1/2} \,. \tag{2.36}$$

Thus,

$$q_i = \frac{(2\pi)^{1/2} M_{11}^{3/2} V_{3,i}}{V_7^{1/2}} \quad (i \leq N_3) \,, \qquad q_{N_3+1} = \frac{(2\pi)^{1/2}}{M_{11}^{3/2} V_7^{1/2}} \,. \tag{2.37}$$

Note that $q_i^2 = 2\kappa_4^2 \tau_i^2$ for all i.

If the radii are appropriately incommensurate then so are the charges. However, the degeneracy D is still nontrivial, 2^J, from $n_i \to -n_i$ for each i (note that in the $J = 100$ model this reduces $q_i^{1/2}$, but only by $\sqrt{2}$). This can be reduced to $D = 2$ by skewing the torus, which couples the different n_i. However, if the stabilization respects the symmetries of the torus there will be an even larger degeneracy: permutations of the axes, obviously, and much more — the full $E_{7(7)}$ U-duality [28]. The resulting D could significantly change the density of levels. A less symmetric compactification will have a much smaller duality group, but we do not know how to estimate a reasonable degeneracy. This effect becomes less important with fewer fluxes.

2.6 Small charges from large dimensions

From eq. (2.24) one finds that a Planck-size cosmological constant can be cancelled in a model with $j = 100$ types of membranes with $q_i^{1/2}$ of order 1/6 in Planck units. If only a few four-forms are present, much smaller charges will be required. For examples, with $j = 6$ and D small one would need $q_i^{1/2} \sim 10^{-10}$ in Planck units. However, these are not small quantities compared to other numbers in elementary particle physics. Indeed, small membrane charges can be related to the gauge hierarchy, if the latter arises by confining the gauge fields to a three-brane living in eleven dimensions and taking some of the extra dimensions to be large, as proposed in ref. [12].

The large internal dimensions will play a double role here. They are the origin of the gauge hierarchy. But in addition, they will lead to small membrane charges, if membranes arise by wrapping a five-brane as described in the previous subsection. This amounts to reducing the gauge hierarchy problem and the cosmological constant problem to the single problem of stabilizing large radii.

In such models the fundamental scale M_{11} is assumed to be near a TeV. The reduction $(2\kappa_4^2)^{-1} = 2\pi M_{11}^9 V_7$ then determines V_7 to be large in fundamental units. For illustration, consider again the seven-torus, with k large dimensions of size

$$2\pi r_l = \frac{1}{M_{11}}(V_7 M_{11}^7)^{1/k}\,, \qquad l = 1, \ldots, k\,, \tag{2.38}$$

and $7 - k$ dimensions of radius 1 in fundamental units:

$$2\pi r_l = \frac{1}{M_{11}}, \quad l = k+1, \ldots, 7\,. \tag{2.39}$$

(In general, of course, the radii could have a range of different sizes. It is trivial to extend this discussion accordingly.) Recalling the charges

$$q_i = \frac{(2\pi)^{1/2} M_{11}^{3/2} V_{3,i}}{V_7^{1/2}} \ \ (i < J)\,, \qquad q_J = \frac{(2\pi)^{1/2}}{M_{11}^{3/2} V_7^{1/2}}\,, \tag{2.40}$$

it is most favorable to consider only q_J plus those q_i for which all the dimensions are small. For these, of which there are $J_0 = \binom{7-k}{3}$ the charges $q_i = q_J$.

We will consider a more general compactification with the same number and sizes of dimensions, but not be restricted by the J_0 attainable on the torus. The condition (2.24) that the charges q_i allow for a sufficiently dense spectrum for λ becomes

$$\frac{\omega_{J'-1}}{D}|2\lambda_{\rm bare}|^{J'/2-1}\Delta\lambda \gtrsim \prod_{i=1}^{J} q_i = (2\pi)^{J'/2} M_{11}^{-3J'/2} V_7^{-J'/2} = (2\pi)^{J'/2} M_{11}^{3J'} \kappa_4^{J'}\,, \tag{2.41}$$

where $J' \equiv J_0 + 1$.

What is the bare cosmological constant in models with large extra dimensions? It receives contributions from the tension of the three-brane, $\lambda_{\rm brane}$, and from the bulk vacuum energy, $\lambda_{\rm bulk}$ (as usual, all contributions from quantum field theory are taken to be subsumed in these quantities) [26, 27]:

$$\lambda_{\rm bare} = \lambda_{\rm brane} + V_7 \lambda_{\rm bulk}\,. \tag{2.42}$$

The most natural value for the brane tension is

$$\lambda_{\rm brane} \sim 2\pi M_{11}^4\,. \tag{2.43}$$

(This value does not follow uniquely from the fundamental theory. The factor of 2π has been included to mimic the form of the M2- and M5-brane tensions.) It is natural (but not necessary) to assume that $\lambda_{\rm bulk}$ is generated by supersymmetry breaking on the brane. This suppresses the vacuum energy by a factor of the compact volume: $\lambda_{\rm bulk} \sim 2\pi M_{11}^4/V_7$, so that both terms in eq. (2.42) will be of order $2\pi M_{11}^4$. (Indeed, the cosmological constant problem in these models amounts to the assumption that the two terms cancel — an assumption that obviously will not be made here.) Supersymmetry breaking in the bulk could lead to a higher value for $|\lambda_{\rm bulk}|$; ultimately, the only constraint comes from bulk stability [27], which is weaker. Recall however that the cancellation mechanism becomes more accurate, the *larger* the magnitude of $\lambda_{\rm bare}$. We can therefore work with the value of eq. (2.43).

The condition on the charges now becomes

$$(2^{-1/2}\kappa_4 M_{11})^{J'+4} \lesssim 10^{-120} \frac{\omega_{J'-1}}{\pi D}\,. \tag{2.44}$$

For the extreme low-dimension picture, where M_{11} is of order 1 TeV, this allows the very modest value $J' = 4$, independent of the number k of large dimensions. This assumes that D is not enormous, as is reasonable for a small value of J'. If we increase J' to 5 then M_{11} can increase to 30 TeV.

If we take the value $\kappa_4 M_{11} \sim 10^{-1.5}$ that is appropriate to the Witten GUT scenario [29], then we need a large number of fluxes, again of order 100 (the precise number is sensitive to uncertain numerical factors, for example in $\lambda_{\rm bare}$). Note also that this requires a cosmological constant of order the GUT scale; a weak-scale cosmological constant cannot be cancelled by our mechanism in this case.

3. Cosmology

In the previous section we showed that multiple four-form strengths arise in most M-theory compactifications, and that these could lead to a spectrum of effective cosmological constants sufficiently finely spaced that some would lie in the observational range. We must now ask why the cosmological constant that we see actually takes such a small value.

3.1 The Brown-Teitelboim mechanism

There are two possible approaches. One could attempt to use the framework of quantum cosmology to argue that the universe was created with λ equal to the smallest positive value in the spectrum [5, 18, 17]. The other possibility is to identify a dynamical mechanism by which an appropriate value of λ is obtained.

We will not follow the quantum cosmology approach. It has the disadvantage that the creation of a space-time from nothing (as opposed to the quantum creation of objects on a given background) is not well understood and possibly ill-defined. The wave-function of Hartle and Hawking [30] would indeed be sharply peaked at the smallest possible value for the cosmological constant. But this would include the effective cosmological constant from any inflaton potential $V(\phi)$, so that there would not be any period of inflation in generic models. The proposals of Linde [31] and Vilenkin [32], on the other hand, would give preference to a large effective cosmological constant, which could come from any combination of contributions from the four-forms and the inflaton. To cancel the cosmological constant one would then need a dynamical effect anyway.

Thus we will employ a dynamical mechanism based on the creation of membranes. This is the approach followed by Brown and Teitelboim (BT) [10, 11], who considered the first model discussed in section 2, with a bare cosmological constant and a single four-form field strength. We will review the dynamics of this case before we generalize the mechanism to multiple four-forms.

BT take λ_{bare} to be negative and n large and positive, so that $\lambda > 0$. Thus the universe will initially be described by de Sitter space. On this background, membrane bubbles can nucleate spontaneously. They appear at a critical size and then expand. This is a non-perturbative quantum effect. Its semi-classical amplitude can be estimated from the euclidean action of appropriate instanton solutions [10, 11]. Inside the membrane, the value of n will be lower or higher by 1, and correspondingly the cosmological constant changes by $(\pm n + 1/2)q^2$.

Increase of n occurs though a dominantly gravitational instanton. It has no equivalent in non-compact spaces, as follows immediately from energy conservation. The instanton for a decreasing cosmological constant is similar to the Coleman instanton for false vacuum decay in flat space [33], with a small correction from gravity. Consequently, the amplitude for increasing the cosmological constant is vastly more suppressed than that for decrease, and one may neglect increase.

Starting from a generic, large value of λ, repeated membrane creation thus produces de Sitter regions with smaller and smaller cosmological constant. The nucleation rate decreases with λ. For a certain range of parameters, membrane creation becomes infinitely suppressed by gravitational effects [34] once λ is no longer positive.

The BT process is analogous to the neutralization of an electric field (an $\mathbf{F_2}$) wrapped around a circle in a $(1+1)$-dimensional world. The Schwinger pair creation of (zero-dimensional) charged particles decreases the field until it has too little energy to nucleate another pair.

BT identified two problems with this scenario. One is the 'empty universe problem', which we will address in section 3.2. The other is the 'gap problem', that in order for some values of the cosmological constant to lie within the observational window one needs membrane charges that are enormously small compared to the ordinary scales of microphysics. We have shown that this problem may be absent in a theory with multiple four-forms.

Let us therefore extend the BT mechanism to the case of J four-forms. In the simplest models one can take $n_i \geq 0$ without loss of generality. For the initial configuration $(n_{1,\text{initial}}, \ldots, n_{J,\text{initial}})$ we only need to assume that the corresponding cosmological constant is positive:

$$\lambda_{\text{bare}} + \frac{1}{2}\sum_{i=1}^{J} n_{i,\text{initial}}^2 q_i^2 > 0\,. \tag{3.1}$$

This condition is generic, in the sense that it excludes only a finite number of configurations. In particular, if the unification scale is high and $J \sim 100$, the charges q_i can be large ($\sim 10^{-1}$) and the inequality will be satisfied with $n_{i,\text{initial}} \sim 1$. If the unification scale is lower, the initial fluxes must be greater. But in such models large fluxes are often needed in any case to stabilize the internal dimensions.

It will be convenient to assume the slightly stronger initial condition that $n_{i,\text{initial}} \geq n_{i,\text{obs}}$ for all i, where $(n_{1,\text{obs}}, \ldots, n_{J,\text{obs}})$ denotes some configuration that lies in the observational window. This will permit us to neglect the strongly gravitational instantons responsible for increasing the n_i.

On the initial de Sitter background, J different types of membranes can be nucleated through appropriate BT instantons. Inside a membrane of the i'th type, the flux n_i is lowered by 1, and the cosmological constant is lowered by $(n_i - 1/2)q_i^2$. Although membranes expand at the speed of light, they typically never collide [35], because they are embedded in de Sitter space and cannot catch up with its expansion. Thus the ambient de Sitter space perdures eternally, harboring all types of membranes for which $n_i > 0$. The same applies iteratively to the de Sitter regions with lower cosmological constant within each bubble. Thus, all combinations $(n_1, \ldots, n_J)$ with $n_i \leq n_{i,\text{initial}}$ and $\lambda > 0$ are attained, including those with λ in the observational range.

In the grid picture, figure 1, the initial configuration corresponds to a grid point some distance outside the sphere, in the $n_i > 0$ quadrant. When a membrane of the i'th type is nucleated, the configuration in its interior corresponds to the neighboring grid point in the negative n_i direction. Nested membranes correspond to a random walk in the grid. Since each membrane bubble harbors all other types of membranes (at least as long as $\lambda > 0$), all such paths through the grid are realized in the universe. Overall, the membrane dynamics corresponds to diffusion through the grid. Every point is populated via many different paths.

3.2 The empty universe problem

3.2.1 Inflation and reheating

The BT process involves spontaneous membrane nucleation in a prolonged de Sitter phase. One would expect this to lead to an empty universe. Particles are produced when a slow-roll field reaches a minimum of its potential and starts to oscillate. By this process, known as reheating, slow-roll inflation avoids the empty universe problem of old inflation. Because membrane nucleation is highly suppressed, it takes an exponentially long time to attain a suitable flux configuration. All fields will reach their vacua long before. Particles may be produced, but they will be wiped away by the remaining phase of the de Sitter expansion. In the end, it appears, we may have achieved too much of nothing: a (nearly) vanishing cosmological constant, but also vanishing entropy.

We will now discuss how this problem may be resolved. If any slow-roll field exists at all, one can argue that the problem does not occur in multiple four-form models with unification at the GUT scale or above, because the high temperature of de Sitter space before the final membrane nucleation kicks the inflaton out of its minimum. Moreover, if the inflaton potential contains a false vacuum, inflation and reheating can also occur in multiple four-form models with low unification scale.

With an inflaton field included, the effective cosmological constant is given by

$$\lambda_{\text{eff}}(\phi) = \lambda_{\text{bare}} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2 + V(\phi)\,. \tag{3.2}$$

We take the inflaton potential $V(\phi)$ to have a stable minimum at $\phi = 0$. By absorption into λ_{bare} we can arrange $V(0) = 0$. The criterion for a suitable configuration $(n_1, \ldots, n_J)$ is that the cosmological constant be small for $\phi = 0$: $\lambda_{\text{eff}}(0) = \lambda \approx 0$. For $\phi \neq 0$ one obtains a positive effective cosmological constant, $\lambda_{\text{eff}}(\phi) = V(\phi)$, at least temporarily.

Slow-roll inflation with $\lambda = 0$ is described as follows. An inflaton field ϕ rolls down in a potential $V(\phi)$. During this time, the universe expands exponentially, like de Sitter space with an effective cosmological constant $\lambda_{\text{eff}}(\phi) = V(\phi)$. Quantum fluctuations during this era freeze when they leave the horizon, forming seeds for density perturbations. When ϕ reaches the bottom of the potential, it oscillates, inflation ends, and the universe is reheated.

The inflaton potential must be very flat. We will not address the difficult problem of how such potentials may arise from a fundamental theory; we note merely that

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they must exist if inflation is the correct explanation for homogeneity and density perturbations. For inflation to last long enough, one must require that the initial value of the inflaton field, ϕ_0, is sufficiently far from the minimum of $V(\phi)$:

$$\phi_0 \geq \phi_* \,, \tag{3.3}$$

where ϕ_* corresponds to sixty e-foldings of de Sitter-like expansion. This is realized, for example, if the inflaton field is initially trapped in a false vacuum far enough from the true minimum at $\phi = 0$. More generically, suitable domains will exist if one assumes chaotic initial conditions in the early universe [36]. In the scenario we have described, however, one must worry that any inflaton field will reach its minimum when λ is still large, because membrane creation takes an exponentially long time. Consequently, ϕ would no longer be available to perturb and reheat the universe when the flux configuration corresponding to $\lambda \approx 0$ is reached.

3.2.2 Kicking the inflaton

The evolution of ϕ is actually a combination of classical slow-roll and brownian motion [37, 38]. The latter can be understood as a random walk induced by the Gibbons-Hawking temperature [39] of de Sitter space:

$$T(\phi) = \frac{H(\phi)}{2\pi} \,, \tag{3.4}$$

where the Hubble parameter is given by[7]

$$H(\phi)^2 = \frac{\lambda_{\rm eff}(\phi)}{3M_{\rm Pl}^2}. \tag{3.5}$$

The characteristic time scale in de Sitter space is the Hubble time, $\Delta t = H^{-1}$. A typical quantum fluctuation of the field ϕ, during the time Δt, is given by [40]

$$|\delta\phi| = \sqrt{2}T(\phi) = \frac{H(\phi)}{\sqrt{2}\pi} \,. \tag{3.6}$$

The classical decrease $|\Delta\phi|$ of the inflaton field can be estimated from the restoring force, $-V'(\phi)$:

$$|\Delta\phi| \approx \frac{1}{2}V'(\phi)(\Delta t)^2 = \frac{V'(\phi)}{2H(\phi)^2} \,. \tag{3.7}$$

A prime denotes differentiation with respect to ϕ. We neglect velocity effects because they are small during slow-roll and average to zero in brownian motion. The random walk dominates over classical evolution if $|\delta\phi| > |\Delta\phi|$, or

$$\sqrt{\frac{2}{3}}\frac{1}{3\pi}\,\lambda_{\rm eff}(\phi)^{3/2}M_{\rm Pl}^{-3} > V'(\phi)\,. \tag{3.8}$$

[7] We use the 'reduced' Planck mass, $M_{\rm Pl} = (8\pi G_{\rm N})^{-1/2} = \kappa_4^{-1} = 2.43 \cdot 10^{18}$ GeV.

We are interested in the temperature of the universe just before a final membrane is nucleated. Consider a flux configuration $(n_1, \ldots, n_{j-1}, n_j + 1, n_{j+1}, \ldots, n_J)$, where $(n_1, \ldots, n_{j-1}, n_j, n_{j+1}, \ldots, n_J)$ corresponds to a cosmological constant in the observational window. If the charges q_i are large, the penultimate cosmological constant,

$$\lambda_{\rm pen} = \left(n_j - \frac{1}{2}\right) q_j^2 \,, \tag{3.9}$$

will be large. Since $\lambda_{\rm eff,pen}(\phi) \geq \lambda_{\rm pen} > 0$, eq. (3.8) will be satisfied for a range of values of ϕ including $\phi = 0$. Therefore the inflaton will take random values within a (finite or infinite) neighborhood of $\phi = 0$.

When the final membrane is nucleated, the temperature in its interior suddenly vanishes. Then the inflaton no longer experiences significant brownian motion and rolls to its minimum. The question is whether this period of inflation is sufficient. (Less ambitiously, one could ask only whether ϕ will be large enough to reheat the universe.) One would like the width of the random distribution of ϕ to be a few times larger than ϕ_*, the value required for 60 e-foldings. Then it will be likely that $\phi \geq \phi_*$ at the time of the final membrane nucleation.

From eqs. (2.31), (2.39), and (2.40) one obtains

$$q_j^2 \approx 8\pi^2 \left(\frac{M_{11}}{M_{\rm Pl}}\right)^6 M_{\rm Pl}^4 \,. \tag{3.10}$$

The inequality, eq. (3.8), will be satisfied if

$$(2n_j - 1)^{3/2} \sqrt{\frac{2}{3}} \frac{8\pi^2}{3} \left(\frac{M_{11}}{M_{\rm Pl}}\right)^9 M_{\rm Pl}^3 > V'(\phi) \,. \tag{3.11}$$

One may take as examples the polynomial potentials $V(\phi) = 10^{-12} M_{\rm Pl}^2 \phi^2/2$ and $V(\phi) = 10^{-14}\phi^4/4$, and note that $\phi_* \approx 15 M_{\rm Pl}$ in both cases. Then eq. (3.8) must be satisfied with $V' \gtrsim 10^{-10} M_{\rm Pl}^3$ for sufficient inflation ($\phi > \phi_*$) to be likely. The condition becomes

$$\frac{M_{11}}{M_{\rm Pl}} \gtrsim (2n_j - 1)^{-1/6} \cdot 10^{-1.3} \,. \tag{3.12}$$

Thus the universe will undergo a normal period of inflation if the unification scale is 10^{17} GeV or higher.

The n_j-dependent factor does not contribute much since J is large and the flux numbers will be of order one. Because of the $(M_{11}/M_{\rm Pl})^9$ suppression in eq. (3.11), this mechanism rapidly becomes inefficient for lower unification scale.

3.2.3 Trapping the inflaton

There is an alternative approach to the empty universe problem. It is less generic, but has the advantage that it can work in models with low unification scale, $M_{11} \geq 1$ TeV. Assume that the potential of the inflaton field (or of any other field with suitable coupling to the inflaton) contains a false vacuum. During the long de Sitter era before the $\lambda \approx 0$ flux configuration is attained, this vacuum will of course decay by Coleman-De Lucia tunnelling [34]. As we discussed earlier, Guth and Weinberg have shown that bubbles do not percolate in de Sitter space [35]. Because the bubbles do not all collide, we need not fear that the entire universe will be converted to the true vacuum. In the ambient metastable space, the BT mechanism proceeds as before. Eventually, it produces regions where $\lambda \approx 0$ while $\lambda_{\rm eff}$ is given by the energy density of the false vacuum. Only then will we be interested in the decay of the false vacuum. After tunnelling, the field emerges on the other side of the barrier. For a wide class of potentials, the field configuration at this point will still be far from the true vacuum. The fields can then roll to the minimum, thus inflating and reheating the universe.

The understanding of the cosmology of models with large internal dimensions is still being developed. This makes a detailed implementation of the generalized BT mechanism difficult. We should caution that there are important constraints on inflationary models that operate after radius stabilization [41].

In this subsection it has been assumed that the effective potential $V(\phi)$ is the same before and after the final membrane nucleation. However, there can be important corrections from the high temperature before the final transition [42]. They will typically make the potential steeper but also shift its minima. Thus, after the final nucleation, the inflaton field will not be in a local minimum and can roll down. Moreover, one would expect coupling constants of the effective field theory to depend on the fluxes. This also contributes to the flux-dependence of the effective potential and generically to a shift of its minima when a membrane is nucleated.[8]

3.3 Vacuum selection

In the BT scenario the universe generically develops a large number of different, exponentially large regions with every value of the cosmological constant in the discretuum, including large values. Why are we located in one of the regions with a small cosmological constant?

Most regions will not contain structure such as galaxies. Observers are necessarily located where structure does form, which restricts us to regions in the *Weinberg window* [43],

$$-10^{-120} M_{\rm Pl}^4 < \lambda < 10^{-118} M_{\rm Pl}^4 \,. \tag{3.13}$$

The upper bound is about 100 times larger than the observed λ. It is obtained by demanding that the cosmological constant must not dominate the evolution of the universe before a redshift of about 4, so that gravitational clustering operates long enough for galaxies to form. The lower bound follows because the universe must not recollapse while stars and galaxies form. Its magnitude is comparable to the observed cosmological constant, but it has opposite sign. Much work has

[8] We thank L. Susskind and S. Thomas for pointing this out to us.

been devoted to strengthening these constraints by more careful astrophysical and statistical arguments (see, e.g., [44, 45, 46, 47] and references therein).

Such considerations may be distasteful to some but should not be viewed as an easy fix. They cannot be applied unless the fundamental theory satisfies a number of rather non-trivial conditions: it must admit different values of the cosmological constant; they must contain at least one value in the observational range; and there must be a dynamical mechanism that allows some regions to attain such a value. The aim of this paper has been to present evidence that all of these conditions may be satisfied in compactified 11D supergravity.

3.4 Stability

In order for our picture to be satisfactory we need the rate of bubble nucleation from a phase with small cosmological constant to be small on the scale of the age of our universe. The tunnelling amplitude is proportional to e^{-B}, where B is the normalized action of the corresponding instanton [10, 11]. A sufficient condition is $B \gtrsim 10^3$.

We consider a single membrane, which changes the flux j from n_j to $n_j - 1$. The domain wall tension is τ_j, given in eq. (2.32), and the change in the cosmological constant is

$$\delta\lambda = -\left(n_j - \frac{1}{2}\right) q_j^2 = -2M_{\rm Pl}^{-2}\left(n_j - \frac{1}{2}\right)\tau_j^2 \,. \tag{3.14}$$

For $n_j \gg 1$, gravity has negligible effects and the action is given by

$$B = \frac{27\pi^2}{2\,(n_j - 1/2)^3\,(2M_{\rm Pl}^{-2} q_j)^2} \,. \tag{3.15}$$

To estimate the n_i we assume approximate equipartition of the energy among the fluxes so that

$$\frac{n_i^2 q_i^2}{2} \approx \frac{2\pi M_{11}^4}{J} \,. \tag{3.16}$$

For the nonzero fluxes in section 2.6, $\tau_i = 2\pi M_{11}^3$, and $q_i^2 = 8\pi^2 (M_{11}/M_{\rm Pl})^2 M_{11}^4$. We obtain

$$B \approx \frac{27\pi^{3/2} J^{3/2}}{16\sqrt{2}(M_{11}/M_{\rm Pl})^3} \,. \tag{3.17}$$

In the large dimension case B is of order 10^{46} and so the tunneling is negligible. Even for the Witten GUT scenario, where $J \sim 100$, it is of order 10^8 and again tunneling is negligible.

At higher unification scales, for which $M_{11}/M_{\rm Pl} > 10^{-1.5}$, one finds $J > 100$. Then eq. (3.16) yields $n_i < 1$ and thus breaks down. Almost all relevant configurations will have $n_i \in \{0, 1\}$ for all $i \in \{1, \ldots, J\}$. We can therefore assume $n_j = 1$. In

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this case the additional suppression due to gravity is significant, although it is never total for our parameters. One finds

$$B = \frac{1728\pi^2}{(2M_{\rm Pl}^{-2} q_j)^2} = 54 \left(\frac{M_{11}}{M_{\rm Pl}}\right)^{-6} . \tag{3.18}$$

Tunnelling will be negligible for $M_{11}/M_{\rm Pl} < 0.6$. Therefore vacuum stability is not a significant constraint on our mechanism. A stronger constraint on $M_{11}/M_{\rm Pl}$ is obtained from eq. (2.41) by requiring a realistic number of three-cycles, say $J < 10^3$.

4. Conclusions

Compactifications of M-theory generally give rise to multiple four-form field strengths. We showed that such theories have vacua with discrete but closely spaced values for the cosmological constant. In the Witten GUT scenario, the spectrum will contain values of λ in the observable range if the number of four-forms is of order 100. (This requires that the cosmological constant to be cancelled is of GUT scale, not weak scale). In models with large internal dimensions, four or five four-forms suffice, and a weak-scale cosmological constant can be cancelled. By repeated membrane nucleation, flux configurations with $\lambda \approx 0$ arise dynamically from generic initial conditions. We argued that entropy and density perturbations can be generated in such regions, and showed that the amplitude for the decay of the $\lambda \approx 0$ vacuum is negligible.

An attractive feature of this proposal is that it simultaneously addresses two questions that are usually treated as distinct. The first question is: why is the cosmological constant not huge? One would expect a vacuum density λ of order $M_{\rm Pl}^4$, or at least TeV^4 with supersymmetry. Until recently this was the only cosmological constant problem. It appeared to require a symmetry ensuring the exact cancellation of all contributions to the cosmological constant. This is difficult because contributions are expected to come from many different scales. The second question is: Why is the cosmological constant not zero? Recent evidence[9] points to a flat universe with $\Omega_{\rm m} \approx 0.3$ and $\Omega_\lambda \approx 0.7$. The favored value for the vacuum energy is $\lambda \approx 10^{-120} M_{\rm Pl}^4 \approx (0.003 \text{ eV})^4$. In particular, a flat universe with vanishing vacuum energy has been ruled out. But if it is difficult to explain $\lambda = 0$, a small non-zero cosmological constant seems to pose an even greater theoretical challenge. The mechanism we propose has limited accuracy because of flux quantization, so that a residual cosmological constant is inevitable.

Our proposal has certain features of the Brown-Teitelboim idea, and also certain features of eternal inflation [38]. Previously, however, both of these ideas have been difficult to realize with a plausible microphysics. Our proposal allows both to be realized within string theory. For the Brown-Teitelboim idea, the main problem

[9] A review of these observations can be found in ref. [48].

was the very small energy scale needed in the discretuum; we see that this can be obtained from a normal hierarchy with multiple fluxes. Eternal inflation with generic polynomial potentials requires scalar field expectation values strictly larger than the Planck scale. In string theory the scale of the field manifold is the string scale, which is no larger than the Planck scale. The manifold is actually noncompact, but the asymptotic regions generally correspond to decompactification of spacetime, and in this region the effective potential generally ceases to be flat. We have realized a version of eternal inflation that does not require such a large scalar, and uses elements already present in string theory.[10] Moreover, if the membrane charges are large, the high temperature of de Sitter space before the final membrane nucleation induces brownian motion of the inflaton field, thus preparing suitable initial conditions for chaotic inflation after the transition.

The main problem with realizing our picture is the stabilization of the compact dimensions, which is of course a ubiquitous problem in string theory. A positive bulk cosmological constant is a useful ingredient [26, 27], but it is not clear that this can be realized in string theory.

It is interesting that the naked singularity proposal [8, 9] appears to lead in the end to a very similar picture. The free parameters that correspond to boundary conditions at a naked singularity in a compact space will become, in a four-dimensional effective lagrangian, variable coupling constants. In the original proposal these were assumed to be continuous and constant in time, but in ref. [50] it was argued that they are discrete and can change across a domain wall, just as for the fluxes considered here. In the example [50] there was a potentially large number of states, of order $e^{\sqrt{N}}$ where N is at Ramond-Ramond charge of the singularity. Note, however, that a charge of order 10^5 is needed to produce a discretuum sufficiently dense to account for the smallness of the cosmological constant. In ref. [50] the main focus was on supersymmetric states, which were all degenerate, but with supersymmetry breaking there will again be a spectrum for λ. Again, stabilization will be an issue.

The appearance of the anthropic principle, even in the weak form encountered here, is not entirely pleasant, but we would argue that it is necessary in any approach where the cosmological constant is a dynamical variable. That is, a small value for the present cosmological constant cannot be obtained by dynamical considerations alone. The point is that we can follow cosmology at least back to nucleosynthesis, when the present cosmological constant contributed only a fraction 10^{-30} to the energy density of the universe, and so was dynamically irrelevant. At earlier times, including the point where the cosmological constant is to have been determined, the fraction would have been even smaller.[11]

[10] A precursor to the idea of four-form-driven eternal inflation was presented in ref. [49].

[11] One exception is the wormhole idea [51], where the value of the cosmological constant in our universe is determined by the presence of other, empty, universes. At least one of the authors retains a certain wary fondness for this possibility.

Acknowledgments

We would like to thank N. Arkani-Hamed, S. Dimopoulos, S. Kachru, A. Linde, J. March-Russell, J. Rahmfeld, L. Susskind, S. Thomas, N. Toumbas, and E. Witten for useful discussions. We would also like to thank J. Feng, J. March-Russell, S. Sethi, and F. Wilczek for informing us about their forthcoming work. While completing this work we learned that J. Donoghue was also pursuing the idea of anthropic determination of the cosmological constant with nondynamical four-forms [52]. The work of J.P. was supported by National Science Foundation grants PHY94-07194 and PHY97-22022. The work of R.B. was supported by a BASF fellowship of the German National Scholarship Foundation.

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